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A Loan Payment Model with Rhythmic Skips

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Abstract

Repayments of loans granted by banks to customers are usually in equal installments. The general formulae of the amount of financial installment, the number of installment, the interest rate of installments, etc. could be derived as considering the sum of present value of repayments made by the customer (installments) to be equal to the present value of the loan. Presenting the different options instead of only fixed installments to customers for repayments of loan installments is very important in terms of reaching more customers. Customers could have some difficulties to pay the loan due to the increased costs in some periods. Therefore, repayments could not be done in these periods. This situation was addressed by Formato (1992) first time and it was called as skip loan payment model.

Formato's (1992) model is improved by Moon as a repayment installments model in a geometric-gradient series. Eroglu and Karaoz (2002) extended Formato's result to the case that periodic payments occur in a linear-gradient series. In this study, general formulae are derived for loan payment models including rhythmic skips with split geometric constant and alternating installments instead of random skips with geometric constant and alternating installments. Examples of the developed model are provided for better understanding and for future research areas. Since prospective customers of financing institutions demand more alternatives for payment plans for their loans, financing institutions need different installment plans. Therefore, it is expected that one of those alternatives would be in this study.

Keywords: Loan payment, Rhythmic skips, Split geometric constant, Installments, Formato

1. INTRODUCTON

The problem of repayment installments of a loan is based on the present value of the debt to be equal the net present value of debt repayments (Iscil, 1997). The general formulae are derived for loan payment models including rhythmic skips with split geometric constant and alternating installments instead of random skips with geometric constant and alternating installments in different financial mathematics books. Those models are given below:

The loan repayment model with constant installments as follows:

$$d = \frac{pr}{1-R^{-n}}. \tag{1}$$

The loan repayment model with geometric alternating installments as follows:

$$d_k = dG^{k-1}, \quad k = 1, \cdots, n. \tag{2}$$

$$d = \begin{cases} \dfrac{p(r-g)}{1-i^n} & , \quad g \neq r \\ \dfrac{pR}{n} & , \quad g = r \end{cases}. \tag{3}$$

The loan repayment model with arithmetical alternating installment series as follows:

$$d_k = d + (k-1)v \quad , \quad k = 1, \cdots, n \tag{4}$$

$$d = \frac{pr^2R^n + v\left[1 + nr - R^n\right]}{r\left(R^n - 1\right)} \tag{5}$$

(Eroglu 2000).

Where;

d : the installment or the first amount of periodic payment,

d_k : the amount of the installment at the end of k^{th} period,

n : the number of installments,

p : the amount of the loan,

r : the periodic interest rate, R=1+r,

g : the proportional change (geometric) in installment amounts, G=1+g, i=GR^{-1} ,

v : the quantitative change (arithmetic) in installment amounts.

The aforementioned loan repayment models assume that installments are made at the end of each period. Formato (1992) developed an installment model in which cases client does not want to make payments at the end of certain periods that s/he will determine, such as due to vacation expenses. Formato's skip payment model is extended to the case where periodic installments change in geometric sequences by Moon (1994) and is extended to the case where periodic payments change in geometric sequences by Eroglu and Karaoz (2002). Furthermore, Eroglu (2001) developed general formulae for the models of installments with arbitrary skipped partially geometric and installments with partially arithmetic changes. The installments that will not be paid at the end of which period are chosen arbitrarily for the above mentioned four studies.

In this study, general formulae are derived for loan payment models including rhythmic skips with split geometric constant and alternating installments instead of random skips with geometric constant and alternating installments instead of choosing the periods where the installments will not be paid arbitrarily.

2. A LOAN REPAYMENT MODEL WHICH HAS RHYTMIC SKIPS WITH SPLIT GEOMETRIC ALTERNATING INSTALLMENT SERIES

In this model, the installments occur as periods, e.g, monthly, quarterly, etc. Period with repayments is the sequential periods where installments are made. Period without repayments is the sequential periods where installments are not made. The fundamental assumption of the model is that the lengths of periods with repayments are equal to each other (each period has an equal number of repayment period) and also the lengths of periods without repayments are equal among themselves. The adjective rhythmic is added to the model because of this assumption. In previous studies, the lengths of periods with repayments are different. This situation also holds for periods without repayments. Another assumption of the model is that the amounts of installments are forming a geometric series. In other words, installments in a period with repayment are equal to each other and form a geometric alteration in sequential two periods.

The following symbols are used in addition to symbols for the model given earlier.

f : the total number of installments in a period with repayment,

h : the number of unpaid installments in a period without repayment,

M_k : the first period number of an installment in a period with repayment following a period without repayment,

L_{k+1} : the last period number of an installment in a period with repayment following a period without repayment,

d_{kj} : the total amount of installments made at the end of the j^{th} period of a period with repayment following k^{th} period without repayment,

s : the total number of periods without repayments,

n : the duration of loan repayment schedule as the number of periods.

The following expressions can be written, since the number of installments of periods with repayments is equal to each other and the lengths of the periods without repayments are equal:

$$M_k = k\left(f + h\right) + 1, \quad k = 0, \cdots, s$$

$$L_{k+1} = k\left(f + h\right) + f, \quad k = 0, \cdots, s$$

$$n = L_{s+1} = s(f+h) + f$$

The following formula is used for when the installments are split geometric alternating:

$$d_{kj} = dG^k, \quad k = 0, \cdots, s \quad j = M_k, \cdots, L_{k+1} \tag{6}$$

The following formulae are obtained, since the amount present value of the loan is equal to the net present value of the installments (see appendices):

$$p = \sum_{k=0}^{s} \sum_{j=M_k}^{L_{k+1}} d_{kj} R^{-j} = \begin{cases} \dfrac{d\left(1-R^{-f}\right)}{r} \left[\dfrac{\left(GR^{-(f+h)}\right)^{s+1} - 1}{GR^{-(f+h)} - 1} \right] & , \quad G \neq R^{f+h} \qquad (7a) \\ \dfrac{d\,(s+1)\left(1-R^{-f}\right)}{r} & , \quad G = R^{f+h} \qquad (7b) \end{cases}$$

and

$$d = \begin{cases} \dfrac{rp\left(GR^{-(f+h)} - 1\right)}{\left(1-R^{-f}\right)\left[\left(GR^{-(f+h)}\right)^{s+1} - 1\right]} & , \quad G \neq R^{f+h} \qquad (8a) \\ \dfrac{rp}{(s+1)\left(1-R^{-f}\right)} & , \quad G = R^{f+h} \qquad (8b) \end{cases}$$

2.1. Example-1: For the case of $G \neq R^{f+h}$

A car with the cash value of \$ 15000 has been bought with the following conditions: There will be 1 month skip after 3 monthly installments and the loan will be paid within 15 months with a 3 percent increase from one period with repayment to the next. Compute the monthly installments when the monthly interest rate is 1.2%.

The problem data: p = 15000, f = 3, h = 1, s = 3, n = 15, g = 0.03, r = 0.012.

The first installment is calculated as d = 1315.19 by using Equation-8a. The installment plan is given in Table 1.

Table 1. The installment plan for Example-1

Months	Installments	The remaining amount of the debt (\$)
0		15000
1	1315.19	(15000*1.012) – 1315.19 = 13864.81
2	1315.19	(13864.81*1.012) – 1315.19 = 12716.00
3	1315.19	(12716.00*1.012) – 1315.19 = 11553.40
4	0	(11553.40*1.012) – 0 = 11692.04
5	1354.64	(11692.04*1.012) – 1354.64 = 10477.70

6	1354.64	(10477.70*1.012) – 1354.64 = 9248.80
7	1354.64	(9248.80*1.012) – 1354.64 = 8005.14
8	0	(8005.14*1.012) – 0 = 8101.20
9	1395.28	(8101.20*1.012) – 1395.28 = 6803.14
10	1395.28	(6803.14*1.012) – 1395.28 = 5489.50
11	1395.28	(5489.50*1.012) – 1395.28 = 4160.09
12	0	(4160.09*1.012) – 0 = 4210.01
13	1437.14	(4210.01*1.012) – 1437.14 = 2823.39
14	1437.14	(2823.39*1.012) – 1437.14 = 1420.13
15	1437.14	(1420.13*1.012) – 1437.14 = 0

2.2. Example-2: For the case of $G = R^{f+h}$

A car with the cash value of \$ 13000 has been bought with the following conditions: There will be 2 months skip after 3 monthly installments and the loan will be paid within 13 months with a 5.101 percent increase from one period with repayment to the next. Compute the monthly installments when the monthly interest rate is 1%.

The problem data: p = 13000, f = 3, h = 2, s = 2, n = 13, g = 0.05101, r = 0.01.

The first installment is calculated as d = 1473.43 by using Equation-8b. The installment plan is given in Table 2.

Table 2. The installment plan for Example-2

Months	Installments	The remaining amount of the debt (\$)	Months	Installments	The remaining amount of the debt (\$)
0		13000	7	1548.59	6179.17
1	1473.43	11656.57	8	1548.59	4692.37
2	1473.43	10299.71	9	0	4739.30
3	1473.43	8929.27	10	0	4786.69
4	0	9018.57	11	1627.58	3206.98
5	0	9108.75	12	1627.58	1611.47
6	1548.59	7651.25	13	1627.58	0

3. A LOAN REPAYMENT MODEL WHICH HAS RHYTMIC SKIPS WITH CONSTANT INSTALLMENTS

A loan repayment model which has rhythmic skips with split geometric alternating installment series becomes a loan repayment model which has rhythmic skips with constant installments when g=0. Therefore, the equations (7a) and (8a) transform equations (9) and (10), respectively.

$$p = \frac{d\left(1-R^{-f}\right)\left(R^{-(f+h)(s+1)}-1\right)}{r\left(R^{-(f+h)}-1\right)} \tag{9}$$

$$d = \frac{rp\left(R^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left(R^{-(f+h)(s+1)}-1\right)} \tag{10}$$

3.1. Example-3

A car with the cash value of $ 8000 has been bought with the following conditions: There will be 1 month skip after 3 monthly installments and the loan will be paid within 15 months with constant repayments. Compute the monthly installments when the monthly interest rate is 0.9%.

The problem data: p = 8000, f = 3, h = 1, s = 3, n = 15, r = 0.009.

The first installment is calculated as d = 715.61 by using Equation-10. The installment plan is given in Table 3.

Table 3. The installment plan for Example-3

Months	Installments	The remaining amount of the debt ($)	Months	Installments	The remaining amount of the debt ($)
0		8000	8	0	4143.31
1	715.61	7356.39	9	715.61	3464.99
2	715.61	6706.99	10	715.61	2780.56
3	715.61	6051.74	11	715.61	2089.98
4	0	6106.21	12	0	2108.79
5	715.61	5445.55	13	715.61	1412.16
6	715.61	4778.95	14	715.61	709.26
7	715.61	4106.35	15	715.61	0

4. CONCLUSION

The problem of repayment installments of a loan is based on the present value of the debt to be equal the net present value of debt repayments. The difference between loan repayment models is due to the alternation of distribution of installments. The most known and used loan repayment models are models with constant, geometric alternating and arithmetic alternating installment series. It can be convenient for customers not to make installments in some periods because of income variability over time. Using this idea, models with arbitrary skips are studied by Formato (1992), Moon (1994), Eroglu (2001), and Eroglu and Karaoz (2002). It is very important for financial institutions increase in the number of loan repayment models in terms of reaching more customers.

In this study, loan repayment models with skips have some rules (such as repayments for three months after two month skips) instead of installments with arbitrary skips are studied. General formulae are derived for the models have rhythmic skip installments with constant and split geometric alternations, and demonstrated with examples.

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APPENDICES

$$p=\sum_{k=0}^{s}\sum_{j=M_k}^{L_{k+1}} d_{kj}R^{-j}=\sum_{k=0}^{s}\sum_{j=k(f+h)+1}^{k(f+h)+f} dG^kR^{-j}=d\sum_{k=0}^{s}\left(G^k\sum_{j=k(f+h)+1}^{k(f+h)+f}R^{-j}\right)=d\sum_{k=0}^{s}\left[G^k\left(R^{-k(f+h)-1}\left(\frac{\left(R^{-f}-1\right)R}{R^{-1}-1}\right)\right)\right]$$

$$= \frac{-dRR^{-1}\left(R^{-f}-1\right)}{r}\sum_{k=0}^{s}\left(GR^{-(f+h)}\right)^{k} = \frac{d\left(1-R^{-f}\right)}{r}\left[\frac{\left(GR^{-(f+h)}\right)^{s+1}-1}{GR^{-(f+h)}-1}\right]$$

and

$$d = \frac{rp\left(GR^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left[GR^{-(f+h)(s+1)}-1\right]}$$